

Soft Games

Naim Çağman, Irfan Deli

Department of Mathematics, Faculty of Arts and Sciences,
Gaziosmanpaşa University, 60250 Tokat, Turkey
naim.cagman@gop.edu.tr

Department of Mathematics, Faculty of Arts and Sciences,
Kilis 7 Aralık University, 79000 Kilis, Turkey,
irfandeli@kilis.edu.tr

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Abstract

In this work, after given the definition of soft sets and their basic operations we define two person soft games which can apply to problems contain vagueness and uncertainty. We then give four solution methods of the games which are soft saddle points, soft lower and soft upper values, soft dominated strategies and soft Nash equilibrium. We also give an example from the real world which shows that the methods can be successfully applied to a financial problem. Finally, we extended the two person soft games to n-person soft games.

Keyword 0.1 *Soft sets, two person soft games, soft payoff functions, soft dominated strategies, soft lower and soft upper values, soft Nash equilibrium.*

1 Introduction

In 1999, Molodtsov [34] introduced soft set theory for modeling vagueness and uncertainty. In [34], Molodtsov pointed out several directions for the applications of soft sets, such as stability and regularization, game theory and operations research, and soft analysis. After Molodtsov, works on soft set theory has been progressing rapidly. For instance; on the theory of soft sets [2, 8, 14, 30, 29, 30, 31], on the soft decision making [9, 10, 17, 31], on the algebraic structures soft sets [1, 37, 34, 35] are some of the selected works.

Game theory is originally the mathematical study of competition and cooperation. In other words, game theory is a study of strategic decision making [26]. Game theory was introduced in 1944 with the publication of von Neumann

and Morgenstern [35]. They started modern game theory with the two-person zero-sum games and its proof. Game theory is mainly used in many fields such as; economics, political science, psychology and so on [5]. Ferguson [19] present various mathematical models of game theory. Binmore [8] focused the cooperative and noncooperative game theory. Aliprantis and Chakrabarti [4] give games with decision making.

In 1965, Zadeh [40] developed the theory of fuzzy sets that is the most appropriate theory for dealing with uncertainties. In recent years, many interesting applications of game theory have been expanded by embedding the ideas of fuzzy sets. The two person zero-sum games with fuzzy payoffs and fuzzy goals game theory have been studied by many authors (e.g. [6, 6, 11, 7, 23, 25, 26, 39]). The max-min solution with respect to a degree of attainment of a fuzzy goal has also been studied (e.g. [1, 16, 38, 37, 41, 42]). Many study of game theory have been expanded by using the ideas of interval value (e.g. [16, 24, 27]). The theory for linear programming problems with fuzzy parameters is introduced (e.g. [6, 4, 5]).

In the classical and fuzzy games, the payoff functions are real valued, and therefore the solution of such games are obtained by using arithmetic operations. Especially, fuzzy games depend on the fuzzy set that is described by its membership function. It is mentioned in [34], there exists a difficulty to set the membership function in each particular case, and also the fuzzy set operations based on the arithmetic operations with membership functions do not look natural since the nature of the membership function is extremely individual.

In this work, we propose a game model for dealing with uncertainties which is free of the difficulties mentioned above. The proposed new game is called a soft game since it is based on soft sets theory. To construct a soft set we can use any parametrization with the help of words and sentences, real numbers, functions, mappings, and so on. Therefore, payoff functions of the soft game are set valued function and solution of the soft games obtained by using the operations of sets that make this game very convenient and easily applicable in practice.

This work is organized as follows. In the next section, most of the fundamental definitions of the operations of soft sets are presented. In Section 3, we construct two person soft games and then give four solution methods for the games which are soft saddle points, soft lower and soft upper value, soft dominated strategies and soft Nash equilibrium. In section 4, we give an application for two person soft games. In section 5, we give n-person soft games that is extension of the two person soft games. In final Section, we concluded the work.

2 Soft sets

In this section, we present the basic definitions and results of soft set theory [8]. More detailed explanations related to this subsection may be found in [8, 30, 34].

Notion of the soft set theory is first given by Molodtsov [34]. Then the

definition of soft set is modified by Çağman and Enginoğlu [8] as follows.

Definition 2.1 [8] Let U be a universe, E be a set of parameters that are describe the elements of U , and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping

$$f_A : E \rightarrow P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A$$

where f_A is called approximate function of the soft set F_A . Generally, f_A, g_B, h_C, \dots will be used as an approximate functions of F_A, G_B, H_C, \dots , respectively. The value of approximate function $f(x)$ may be arbitrary, some of them may be empty, some may have nonempty intersection.

It is noting that the soft set is a parametrized family of subsets of the set U , and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A .

Note that if $f_A(x) = \emptyset$, then the element $(x, f_A(x))$ is not appeared in F_A .

Example 2.2 Suppose that $U = \{u_1, u_2, u_3, u_4\}$ is the universe contains four cars under consideration in an auto agent and $E = \{x_1, x_2, x_3, x_4\}$ is the set of parameters, where x_i ($i = 1, 2, 3, 4$) stand for ‘safety’, ‘cheap’, ‘modern’ and ‘large’, respectively.

A customer to select a car from the auto agent, can construct a soft set F_A that describes the characteristic of cars according to own requests. Assume that $A = \{x_1, x_2, x_3, x_4\} \subseteq E$ and $f_A(x_1) = \{u_1, u_2\}$, $f_A(x_2) = \{u_1, u_2, u_4\}$, $f_A(x_3) = \emptyset$, $f_A(x_4) = U$ then the soft-set F_A is written by

$$F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2, u_4\}), (x_4, U)\}$$

By using same parameter set A , another customer to select a car from the same auto agent, can construct a soft set G_A according to own requests. Here G_A may be different then F_A . Assume that $g_A(x_1) = \{u_1, u_2\}$, $g_A(x_2) = \{u_1, u_2, u_3\}$, $g_A(x_3) = \{u_1, u_2\}$, $g_A(x_4) = \{u_1\}$ then the soft-set G_A is written by

$$G_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_1, u_2, u_3\}), (x_3, \{u_1, u_2\}), (x_4, \{u_1\})\}$$

Definition 2.3 [8] Let F_A and G_B be two soft sets. Then,

- a) If $f_A(x) = \emptyset$ for all $x \in E$, then F_A is called a empty soft set, denoted by F_\emptyset .
- b) If $f_A(x) \subseteq g_B(x)$ for all $x \in E$, then F_A is a soft subset of G_B , denoted by $F_A \tilde{\subseteq} G_B$.

Definition 2.4 [8] Let F_A and G_B be two soft sets. Then,

- a) Complement of F_A is denoted by F_A^c . Its approximate function f_{A^c} is defined by

$$f_{A^c}(x) = U \setminus f_A(x) \text{ for all } x \in E$$

- b) Union of F_A and G_B is denoted by $F_A \tilde{\cup} G_B$. Its approximate function $f_A \tilde{\cup} g_B$ is defined by

$$(f_A \tilde{\cup} g_B)(x) = f_A(x) \cup g_B(x) \quad \text{for all } x \in E.$$

- c) Intersection of F_A and G_B is denoted by $F_A \tilde{\cap} G_B$. Its approximate function $f_A \tilde{\cap} g_B$ is defined by

$$(f_A \tilde{\cap} g_B)(x) = f_A(x) \cap g_B(x) \quad \text{for all } x \in E.$$

3 Two Person Soft Games

In this section, we construct two person soft games with soft payoffs. We then give four solution methods for the games. The basic definitions and preliminaries of the game theory we refer to [4, 19, 34, 36, 40].

Definition 3.1 Let E be a set of strategy and $X, Y \subseteq E$. A choice of behaviour in a soft game is called an action. The elements of $X \times Y$ are called action pairs. That is, $X \times Y$ is the set of available actions.

Definition 3.2 Let U be a set of alternatives, $P(U)$ be the power set of U , E be a set of strategies, $X, Y \subseteq E$. Then, a set valued function

$$f_{X \times Y} : X \times Y \rightarrow P(U)$$

is called a soft payoff function. For each $(x, y) \in X \times Y$, the value $f_{X \times Y}(x, y)$ is called a soft payoff.

Definition 3.3 Let $X \times Y$ be a set of action pairs. Then, an action $(x^*, y^*) \in X \times Y$ is called an optimal action if

$$f_{X \times Y}(x^*, y^*) \supseteq f_{X \times Y}(x, y) \quad \text{for all } (x, y) \in X \times Y.$$

Definition 3.4 Let $X \times Y$ be a set of action pairs and $(x_i, y_j), (x_r, y_s) \in X \times Y$. Then,

- a) if $f_{X \times Y}(x_i, y_j) \supset f_{X \times Y}(x_r, y_s)$, we says that a player strictly prefers action pair (x_i, y_j) over action (x_r, y_s) ,
- b) if $f_{X \times Y}(x_i, y_j) = f_{X \times Y}(x_r, y_s)$, we says that a player is indifferent between the two actions,
- c) if $f_{X \times Y}(x_i, y_j) \supseteq f_{X \times Y}(x_r, y_s)$, we says that a player either prefers (x_i, y_j) to (x_r, y_s) or is indifferent between the two actions.

Definition 3.5 Let $f_{X \times Y}^k$ be a soft payoff for Player k , ($k = 1, 2$), and $(x_i, y_j), (x_r, y_s) \in X \times Y$. Then, Player k is called rational, if the player's soft payoff satisfies the following conditions:

- a) Either $f_{X \times Y}^k(x_i, y_j) \supseteq f_{X \times Y}^k(x_r, y_s)$ or $f_{X \times Y}^k(x_r, y_s) \supseteq f_{X \times Y}^k(x_i, y_j)$
- b) If $f_{X \times Y}^k(x_i, y_j) \supseteq f_{X \times Y}^k(x_r, y_s)$ and $f_{X \times Y}^k(x_r, y_s) \supseteq f_{X \times Y}^k(x_i, y_j)$, then $f_{X \times Y}^k(x_i, y_j) = f_{X \times Y}^k(x_r, y_s)$.

Definition 3.6 Let X and Y be a set of strategies of Player 1 and 2, respectively, U be a set of alternatives and $f_{X \times Y}^k : X \times Y \rightarrow P(U)$ be a soft payoff function for player k , ($k = 1, 2$). Then, for each Player k , a two person soft game (tps-game) is defined by a soft set over U as

$$F_{X \times Y}^k = \{((x, y), f_{X \times Y}^k(x, y)) : (x, y) \in X \times Y\}$$

The tps-game is played as follows: at a certain time Player 1 chooses a strategy $x_i \in X$, simultaneously Player 2 chooses a strategy $y_j \in Y$ and once this is done each player k ($k=1,2$) receives the soft payoff $f_{X \times Y}^k(x_i, y_j)$.

If $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_n\}$, then the soft payoffs of $F_{X \times Y}^k$ can be arranged in the form of the $m \times n$ matrix shown in Table 1.

$F_{X \times Y}^k$	y_1	y_2	\dots	y_n
x_1	$f_{X \times Y}^k(x_1, y_1)$	$f_{X \times Y}^k(x_1, y_2)$	\dots	$f_{X \times Y}^k(x_1, y_n)$
x_2	$f_{X \times Y}^k(x_2, y_1)$	$f_{X \times Y}^k(x_2, y_2)$	\dots	$f_{X \times Y}^k(x_2, y_n)$
\vdots	\vdots	\vdots	\ddots	\vdots
x_m	$f_{X \times Y}^k(x_m, y_1)$	$f_{X \times Y}^k(x_m, y_2)$	\dots	$f_{X \times Y}^k(x_m, y_n)$

Table 1: The two person soft game

Now, we can give an example for tps-game.

Example 3.7 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $P(U)$ be the power set of U , $E = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of strategies and $X = \{x_1, x_3, x_5\}$ and $Y = \{x_1, x_2, x_4\}$ be a set of the strategies Player 1 and 2, respectively.

If Player 1 constructs a tps-games as follows,

$$F_{X \times Y}^1 = \left\{ ((x_1, x_1), \{u_1, u_2, u_5, u_8\}), (x_1, x_2), \{u_1, u_2, u_3, u_4, u_5, u_8\}), (x_1, x_4), \{u_3, u_8\}), ((x_3, x_1), \{u_1, u_3, u_7\}), (x_3, x_2), \{u_1, u_2, u_3, u_5, u_6, u_7\}), (x_3, x_4), \{u_1, u_2, u_3\}), ((x_5, x_1), \{u_3, u_4, u_5, u_8\}), (x_5, x_2), \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}), (x_5, x_4), \{u_1, u_2, u_3, u_8\}) \right\}$$

then the soft payoffs of the game can be arranged as in Table 2,

$F_{X \times Y}^1$	x_1	x_2	x_4
x_1	$\{u_1, u_2, u_5, u_8\}$	$\{u_1, u_2, u_3, u_4, u_5, u_8\}$	$\{u_3, u_8\}$
x_3	$\{u_1, u_3, u_7\}$	$\{u_1, u_2, u_3, u_5, u_6, u_7\}$	$\{u_1, u_2, u_3\}$
x_5	$\{u_3, u_4, u_5, u_8\}$	$\{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}$	$\{u_1, u_2, u_3, u_8\}$

Table 2

Let us explain some element of this game; if Player 1 select x_3 and Player 2 select x_2 , then the value of game will be a set $\{u_1, u_2, u_3, u_5, u_6, u_7\}$, that is, $f_{X \times Y}^1(x_3, x_2) = \{u_1, u_2, u_3, u_5, u_6, u_7\}$. In this case, Player 1 wins the set of alternatives $\{u_1, u_2, u_3, u_5, u_6, u_7\}$ and Player 2 lost the same set of alternatives.

Similarly, if Player 2 constructs a tps-game as follows,

$$F_{X \times Y}^2 = \left\{ ((x_1, x_1), \{u_3, u_4, u_6, u_7\}), (x_1, x_2), \{u_6, u_7\}), (x_1, x_4), \{u_1, u_2, u_4, u_5, u_6, u_7\}), ((x_3, x_1), \{u_2, u_4, u_5, u_6, u_8\}), (x_3, x_2), \{u_4, u_8\}), (x_3, x_4), \{u_4, u_5, u_6, u_7, u_8\}), ((x_5, x_1), \{u_1, u_2, u_6, u_7\}), (x_5, x_2), \{u_7\}), (x_5, x_4), \{u_4, u_5, u_6, u_7\}) \right\}$$

then the soft payoffs of the game can be arranged as in Table 3,

$F_{X \times Y}^2$	x_1	x_2	x_4
x_1	$\{u_3, u_4, u_6, u_7\}$	$\{u_6, u_7\}$	$\{u_1, u_2, u_4, u_5, u_6, u_7\}$
x_3	$\{u_2, u_4, u_5, u_6, u_8\}$	$\{u_4, u_8\}$	$\{u_4, u_5, u_6, u_7, u_8\}$
x_5	$\{u_1, u_2, u_6, u_7\}$	$\{u_7\}$	$\{u_4, u_5, u_6, u_7\}$

Table 3

Let us explain some element of this tps-game; if Player 1 select x_3 and Player 2 select x_2 , then the value of game will be a set $\{u_4, u_8\}$, that is, $f_{X \times Y}^2(x_3, x_2) = \{u_4, u_8\}$. In this case, Player 1 wins the set of alternatives $\{u_4, u_8\}$ and Player 2 lost $\{u_4, u_8\}$.

Now the two person zero sum game on the classical game theory will be a two person empty intersection game on the soft game theory. It is given in following definition.

Definition 3.8 A tps-game is called a two person empty intersection soft game if intersection of the soft payoff of players is empty set for each action pairs.

For instance, Example 9.7 is a two person empty intersection soft game.

Definition 3.9 Let $f_{X \times Y}^k$ be a soft payoff function of a tps-game $F_{X \times Y}^k$. If the following properties hold

$$a) \bigcup_{i=1}^m f_{X \times Y}^k(x_i, y_j) = f_{X \times Y}^k(x, y)$$

$$b) \bigcap_{j=1}^n f_{X \times Y}^k(x_i, y_j) = f_{X \times Y}^k(x, y)$$

then $f_{X \times Y}^k(x, y)$ is called a soft saddle point of Player k 's in the tps-game.

Note that if $f_{X \times Y}^1(x, y)$ is a soft saddle point of a tps-game $F_{X \times Y}^1$, then Player 1 can then win at least by choosing the strategy $x \in X$, and Player 2 can keep her/his loss to at most $f_{X \times Y}^1(x, y)$ by choosing the strategy $y \in Y$. Hence the soft saddle point is a value of the tps-game.

Example 3.10 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3, x_4\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies for Player 1 and 2, respectively. Then, tps-game of Player 1 is given as in Table 4,

$F_{X \times Y}^1$	y_1	y_2	y_3
x_1	$\{u_2, u_4, u_7\}$	$\{u_4\}$	$\{u_4\}$
x_2	$\{u_5\}$	$\{u_7\}$	$\{u_4, u_7\}$
x_3	$\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$	$\{u_4, u_8\}$	$\{u_7, u_8\}$
x_4	$\{u_2, u_4, u_5, u_7, u_8\}$	$\{u_1, u_4, u_7, u_8\}$	$\{u_4, u_7, u_8\}$

Table 4

Clearly,

$$\begin{aligned} \bigcup_{i=1}^4 f_{X \times Y}^1(x_i, y_1) &= \{u_2, u_4, u_5, u_7, u_8, u_{10}\}, \\ \bigcup_{i=1}^4 f_{X \times Y}^1(x_i, y_2) &= \{u_1, u_4, u_7, u_8\}, \\ \bigcup_{i=1}^4 f_{X \times Y}^1(x_i, y_3) &= \{u_4, u_7, u_8\}, \end{aligned}$$

and

$$\begin{aligned} \bigcap_{j=1}^3 f_{X \times Y}^1(x_1, y_j) &= \{u_4\}, \\ \bigcap_{j=1}^3 f_{X \times Y}^1(x_2, y_j) &= \phi, \\ \bigcap_{j=1}^3 f_{X \times Y}^1(x_3, y_j) &= \{u_8\}, \\ \bigcap_{j=1}^3 f_{X \times Y}^1(x_4, y_j) &= \{u_4, u_7, u_8\}. \end{aligned}$$

Therefore, $\{u_4, u_7, u_8\}$ is a soft saddle point of the tps-game, since the intersection of the forth row is equal to the union of the third column. So, the value of the tps-game is $\{u_4, u_7, u_8\}$.

Note that every tps-game has not a soft saddle point. (For instance, in the above example, if $\{u_4, u_7, u_8\}$ is replaced with $\{u_4, u_7, u_8, u_9\}$ in soft payoff $f_{X \times Y}^1(x_4, y_3)$, then a soft saddle point of the game can not be found.) Saddle point can not be used for a tps-game, soft upper and soft lower values of the tps-game may be used is given in the following definition.

Definition 3.11 Let $F_{X \times Y}$ be a tps-game with its soft payoff function $f_{X \times Y}$. Then,

i. Soft upper value of the tps-game, denoted \overline{v} , is defined by

$$\overline{v} = \bigcap_{y \in Y} (\bigcup_{x \in X} (f_{X \times Y}(x, y)))$$

ii. Soft lower value of the tps-game, denoted \underline{v} , is defined by

$$\underline{v} = \bigcup_{x \in X} (\bigcap_{y \in Y} (f_{X \times Y}(x, y)))$$

iii. If soft upper and soft lower value of a tps-game are equal, they are called value of the tps-game, noted by v . That is $v = \underline{v} = \overline{v}$.

Example 3.12 Let us consider Table 4 in Example 9.10. It is clear that soft upper value $\overline{v} = \{u_4, u_7, u_8\}$ and soft lower value $\underline{v} = \{u_4, u_7, u_8\}$, hence $\underline{v} = \overline{v}$. It means that value of the tps-game is $\{u_4, u_7, u_8\}$.

Theorem 3.13 \underline{v} and \bar{v} be a soft lower and soft upper value of a tps-game, respectively. Then, the soft lower value is subset or equal to the soft upper value, that is,

$$\underline{v} \subseteq \bar{v}$$

Example 3.14 Let us consider soft upper value \bar{v} and soft lower value \underline{v} in Example 9.12. It is clear that $\bar{v} = \{u_4, u_7, u_8\} \subseteq \underline{v} = \{u_4, u_7, u_8\}$, hence $\underline{v} \subseteq \bar{v}$.

Theorem 3.15 Let $f_{X \times Y}(x, y)$ be a soft saddle point, \underline{v} be a soft lower value and \bar{v} be a soft upper value of a tps-game. Then,

$$\underline{v} \subseteq f_{X \times Y}(x, y) \subseteq \bar{v}$$

Corollary 3.16 Let $f_{X \times Y}(x, y)$ be a soft saddle point, \underline{v} be a soft lower value and \bar{v} be a soft upper value of a tps-game. If $v = \underline{v} = \bar{v}$, then $f_{X \times Y}(x, y)$ is exactly v .

Example 3.17 Let us consider Table 4 in Example 9.10 and soft upper value \bar{v} and soft lower value \underline{v} in Example 9.12. It is clear that soft saddle point $f_{X \times Y}(x, y)$ is exactly $v = \underline{v} = \bar{v} = \{u_4, u_7, u_8\}$.

Note that in every tps-game, the soft lower value \underline{v} can not be equals to the soft upper value \bar{v} . (For instance, in the above example, if $\{u_4, u_7, u_8\}$ is replaced with $\{u_4, u_7, u_8, u_9\}$ in soft payoff $f_{X \times Y}^1(x_4, y_3)$, then the soft lower value \underline{v} can not be equals to the soft upper value \bar{v} .) If in a tps-game $\underline{v} \neq \bar{v}$, then to get the solution of the game soft dominated strategy may be used. We define soft dominated strategy for tps-game as follows.

Definition 3.18 Let $F_{X \times Y}^1$ be a tps-game with its soft payoff function $f_{X \times Y}^1$. Then,

- a) a strategy $x_i \in X$ is called a soft dominated to another strategy $x_r \in X$, if $f_{X \times Y}^1(x_i, y) \supseteq f_{X \times Y}^1(x_r, y)$ for all $y \in Y$,
- b) a strategy $y_j \in Y$ is called a soft dominated to another strategy $y_s \in Y$, if $f_{X \times Y}^1(x, y_j) \subseteq f_{X \times Y}^1(x, y_s)$ for all $x \in X$

By using soft dominated strategy, tps-games may be reduced by deleting rows and columns that are obviously bad for the player who uses them. This process of eliminating soft dominated strategies sometimes leads us to a solution of a tps-game. Such a method of solving tps-game is called a soft elimination method.

The following tps-game can be solved by using the method.

Example 3.19 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies for Player 1 and 2, respectively. Then, tps-game of Player 1 is given as in Table 5,

$F_{X \times Y}^1$	y_1	y_2	y_3
x_1	$\{u_2, u_4, u_7\}$	$\{u_4\}$	$\{u_4\}$
x_2	$\{u_5\}$	$\{u_7\}$	$\{u_4, u_7\}$
x_3	$\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$	$\{u_4, u_7, u_8\}$	$\{u_4, u_7, u_8\}$

Table 5

The last column is dominated by the middle column. Deleting the last column we can obtain Table 6 as:

$F_{X \times Y}^1$	y_1	y_2
x_1	$\{u_2, u_4, u_7\}$	$\{u_4\}$
x_2	$\{u_5\}$	$\{u_7\}$
x_3	$\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$	$\{u_4, u_7, u_8\}$

Table 6

Now, in Table 6, the top row is dominated by the bottom row. (Note that this is not the case in Table 5). Deleting the top row we obtain Table 7 as:

$F_{X \times Y}^1$	y_1	y_2
x_2	$\{u_5\}$	$\{u_7\}$
x_3	$\{u_2, u_4, u_5, u_7, u_8, u_{10}\}$	$\{u_4, u_7, u_8\}$

Table 7

In Table 7, Player 1 has a soft dominant strategy x_3 so that x_2 is now eliminated. Player 2 can now choose between y_1 and y_2 and she/he will clearly choose y_2 . The solution using the method is (x_3, y_2) , that is, value of the tps-game is $\{u_4, u_7, u_8\}$.

Note that the soft elimination method cannot be used for some tps-games which do not have a soft dominated strategies. In this case, we can use soft Nash equilibrium that is defined as follows.

Definition 3.20 Let $F_{X \times Y}^k$ be a tps-game with its soft payoff function $f_{X \times Y}^k$ for $k = 1, 2$. If the following properties hold

a) $f_{X \times Y}^1(x^*, y^*) \supseteq f_{X \times Y}^1(x, y^*)$ for each $x \in X$

b) $f_{X \times Y}^2(x^*, y^*) \supseteq f_{X \times Y}^2(x^*, y)$ for each $y \in Y$

then, $(x^*, y^*) \in X \times Y$ is called a soft Nash equilibrium of a tps-game.

Note that if $(x^*, y^*) \in X \times Y$ is a soft Nash equilibrium of a tps-game, then Player 1 can then win at least $f_{X \times Y}^1(x^*, y^*)$ by choosing strategy $x^* \in X$, and Player 2 can win at least $f_{X \times Y}^2(x^*, y^*)$ by choosing strategy $y^* \in Y$. Hence the soft Nash equilibrium is an optimal action for tps-game, therefore, $f_{X \times Y}^k(x^*, y^*)$ is the solution of the tps-game for Player k , $k = 1, 2$.

Following game, given in Example 9.21, can be solved by soft Nash equilibrium, but it is very difficult to solve by using the others methods.

Example 3.21 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2, y_3\}$ be the strategies Player 1 and and 2, respectively. Then, tps-game of Player 1 is given as in Table 8,

$F_{X \times Y}^1$	y_1	y_2	y_3
x_1	$\{u_1, u_2, u_4, u_7, u_8, u_9\}$	$\{u_1, u_2, u_4, u_7, u_8\}$	$\{u_1, u_2, u_3, u_4, u_7, u_8\}$
x_2	$\{u_1, u_2, u_3, u_5\}$	$\{u_1, u_4, u_7, u_8\}$	$\{u_1, u_2, u_3, u_4, u_5, u_7\}$
x_3	$\{u_2, u_5, u_7, u_8, u_{10}\}$	$\{u_2, u_4, u_7, u_8\}$	$\{u_4, u_5, u_7, u_8, u_{10}\}$

Table 8

and *tps*-game of Player 2 is given as in Table 9,

$F_{X \times Y}^2$	y_1	y_2	y_3
x_1	$\{u_3, u_5, u_6, u_{10}\}$	$\{u_3, u_5, u_6, u_9, u_{10}\}$	$\{u_5, u_6, u_9, u_{10}\}$
x_2	$\{u_4, u_6, u_7, u_8, u_9, u_{10}\}$	$\{u_2, u_3, u_5, u_6, u_9, u_{10}\}$	$\{u_6, u_8, u_9, u_{10}\}$
x_3	$\{u_1, u_3, u_4, u_6, u_9\}$	$\{u_1, u_3, u_5, u_6, u_9, u_{10}\}$	$\{u_1, u_2, u_3, u_6, u_9\}$

Table 9

From the tables, we have

a) $f_{X \times Y}^1(x_1, y_2) \supseteq f_{X \times Y}^1(x, y_2)$ for each $x \in X$, and

b) $f_{X \times Y}^2(x_1, y_2) \supseteq f_{X \times Y}^2(x_1, y)$ for each $y \in Y$

then, $(x_1, y_2) \in X \times Y$ is a soft Nash equilibrium. Therefore, $f_{X \times Y}^1(x_1, y_2) = \{u_1, u_2, u_4, u_7, u_8\}$ and $f_{X \times Y}^2(x_1, y_2) = \{u_3, u_5, u_6, u_9, u_{10}\}$ are the solution of the *tps*-game for Player 1 and Player 2, respectively.

4 An Application

In this section, we give a financial problem that are solved by using both soft dominated strategy and soft saddle point methods.

There are two companies, say Player 1 and Player 2, who competitively want to increase sale of produces in the country. Therefore, they give advertisements. Assume that two companies have a set of different products $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ where for $i = 1, 2, \dots, 8$, the product u_i stand for “oil”, “salt”, “honey”, “jam”, “cheese”, “sugar”, “cooker”, and “jar”, respectively. The products can be characterized by a set of strategy $E = \{x_i : i = 1, 2, 3\}$ which contains styles of advertisement where for $j = 1, 2, 3$, the strategies x_j stand for “TV”, “radio” and “newspaper”, respectively.

Suppose that $X = \{x_1, x_2, x_3\}$ and $Y = \{x_1, x_2, x_3\}$ are strategies of Player 1 and 2, respectively. Then, a *tps*-game of Player 1 is given as in Table 10.

$F_{X \times Y}^1$	x_1	x_2	x_3
x_1	$\{u_1, u_2, u_3, u_5, u_8\}$	$\{u_1, u_2, u_3, u_4, u_5, u_8\}$	$\{u_3\}$
x_2	$\{u_1, u_3, u_7\}$	$\{u_1, u_2, u_3, u_5, u_6, u_7\}$	$\{u_2, u_3\}$
x_3	$\{u_1, u_2, u_3, u_4, u_5\}$	$\{u_1, u_2, u_3, u_4, u_5, u_6, u_8\}$	$\{u_1, u_2, u_3\}$

Table 10

In Table 10, let us explain action pair (x_1, y_1) ; if Player 1 select $x_1 = \text{“TV”}$ and Player 2 select $y_1 = \text{“TV”}$, then the soft payoff of Player 1 is a set $\{u_1, u_2, u_3, u_5, u_8\}$, that is, $f_{X \times Y}^1(x_1, y_1) = \{u_1, u_2, u_3, u_5, u_8\}$. In this case, Player 1 increase sale of $\{u_1, u_2, u_3, u_5, u_8\}$ and Player 2 decrease sale of $\{u_1, u_2, u_3, u_5, u_8\}$.

We can now solve the game. It is seen in Table 10,

$$\begin{aligned} \{u_1, u_2, u_3, u_5, u_8\} &\subseteq \{u_1, u_2, u_3, u_4, u_5, u_8\} \\ \{u_1, u_3, u_7\} &\subseteq \{u_1, u_2, u_3, u_5, u_6, u_7\} \\ \{u_1, u_2, u_3, u_4, u_5\} &\subseteq \{u_1, u_2, u_3, u_4, u_5, u_6, u_8\} \end{aligned}$$

the middle column is dominated by the light column. We then deleting the middle column we obtain Table 11.

$F_{X \times Y}^1$	y_1	y_3
x_1	$\{u_1, u_2, u_3, u_5, u_8\}$	$\{u_3\}$
x_2	$\{u_1, u_3, u_7\}$	$\{u_2, u_3\}$
x_3	$\{u_1, u_2, u_3, u_4, u_5\}$	$\{u_1, u_2, u_3\}$

Table 11

In Table 11, there is no another soft dominated strategy, we can use soft saddle point method.

$$\begin{aligned} \bigcup_{i=1}^3 f_{X \times Y}^1(x_i, y_1) &= \{u_1, u_2, u_3, u_4, u_5, u_7, u_8\} \\ \bigcup_{i=1}^3 f_{X \times Y}^1(x_i, y_3) &= \{u_1, u_2, u_3\} \\ \bigcap_{j=2}^3 f_{X \times Y}^1(x_1, y_j) &= \{u_3\} \\ \bigcap_{j=2}^3 f_{X \times Y}^1(x_2, y_j) &= \{u_3\} \\ \bigcap_{j=2}^3 f_{X \times Y}^1(x_3, y_j) &= \{u_1, u_2, u_3\} \end{aligned}$$

Here, optimal strategy of the game is (x_3, y_3) since

$$\bigcup_{i=1}^3 f_{X \times Y}^1(x_i, y_3) = \bigcap_{j=2}^3 f_{X \times Y}^1(x_3, y_j)$$

Therefore, value of the *tps*-game is $\{u_1, u_2, u_3\}$.

5 *n*-Person Soft Games

In many applications the soft games can be often played between more than two players. Therefore, *tps*-games can be extended to *n*-person soft games.

From now on, X_n^\times will be used for $X_1 \times X_2 \times \dots \times X_n$.

Definition 5.1 Let U be a set of alternatives, $P(U)$ be the power set of U , E be a set of strategies, $X_1, X_2, \dots, X_n \subseteq E$, and X_k is the set of strategies of Player k , ($k = 1, 2, \dots, n$). Then, for each Player k , an *n*-person soft game (*nps*-game) is defined by a soft set over U as

$$F_{X_n^\times}^k = \{((x_1, x_2, \dots, x_n), f_{X_n^\times}^k(x_1, x_2, \dots, x_n)) : (x_1, x_2, \dots, x_n) \in X_n^\times\}$$

where $f_{X_n^\times}^k$ is a soft payoff function of Player k .

The nps-game is played as follows: at a certain time Player 1 chooses a strategy $x_1 \in X_1$ and simultaneously each Player k ($k = 2, \dots, n$) chooses a strategy $x_k \in X_k$ and once this is done each player k receives the soft payoff $f_{X_n^\times}^k(x_1, x_2, \dots, x_n)$.

Definition 5.2 Let $F_{X_n^\times}^k$ be an nps-game with its soft payoff function $f_{X_n^\times}^k$ for $k = 1, 2, \dots, n$. Then, a strategy $x_k \in X_k$ is called a soft dominated to another strategy $x \in X_k$, if

$$f_{X_n^\times}^k(x_1, \dots, x_{k-1}, x_k, x_{k+1}, \dots, x_n) \supseteq f_{X_n^\times}^k(x_1, \dots, x_{k-1}, x, x_{k+1}, \dots, x_n)$$

for each strategy $x_i \in X_i$ of player i ($i = 1, 2, \dots, k-1, k+1, \dots, n$), respectively.

Definition 5.3 Let $f_{X_n^\times}^k$ be a soft payoff function of a nps-game $F_{X_n^\times}^k$. If for each player k ($k=1,2,\dots,n$) the following properties hold

$$f_{X_n^\times}^k(x_1^*, \dots, x_{k-1}^*, x_k^*, x_{k+1}^*, \dots, x_n^*) \supseteq f_{X_n^\times}^k(x_1^*, \dots, x_{k-1}^*, x, x_{k+1}^*, \dots, x_n^*)$$

for each $x \in X_k$, then $(x_1^*, x_2^*, \dots, x_n^*) \in X_n^\times$ is called a soft Nash equilibrium of an nps-game.

6 Conclusion

In this paper, we first present the basic definitions and results of soft set theory. We then construct *tps*-games with soft payoffs. We also give four solution methods for the *tps*-games with examples. To applied the game to the real world problem we give an example which shows the methods can be successfully applied to a financial problem. Finally, we extended the two person soft games to n -person soft games. The soft games may be applied to many fields and more comprehensive in the future to solve the related problems, such as; computer science, decision making, and so on.

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has been progressing rapidly and is finding applications in a wide variety of fields such as theory of soft sets (e.g: [2, 8, 15, 30, 37]), soft decision making (e.g: [9, 10, 17, 18, 22, 31]), algebraic structures of soft sets (e.g: [1, 3, 21]), soft topologies (e.g: [12, 32, 36, 38, 41]), fuzzy soft sets (e.g: [13, 14]), and intuitionistic fuzzy soft sets (e.g: [19, 20, 28, 29, 33]).

Game theory was introduced in 1944 with the study of von Neumann and Morgenstern [35] and then they started modern game theory. Game theory has successfully used in logic, decision making process, economics, political science, and computer science, and so on. In recent years, many interesting applications of game theory have been expanded by embedding the ideas of fuzzy sets (e.g. [6, 7, 23, 25, 26, 39]). The game theory have also been expanded by using the ideas of interval data (e.g. [16, 24, 27]). The linear programming problems with fuzzy parameters is introduced (e.g. [4, 5]).

The notion of soft games is given by Çağman and Deli in [11]. In this work, we define a fuzzy soft game for dealing with uncertainties that is based on both soft sets and fuzzy sets. Therefore, payoff functions of the fuzzy soft game are set valued function and solution of the soft games obtained by using the operations of soft sets and fuzzy sets that make this game very convenient and easily applicable in practice.

In this paper is organized as follows. In the next section, the fundamental definition and most of operations of fuzzy soft sets are presented. In Section 3, we construct two person fuzzy soft games and then give four solution methods for the games which are soft saddle points, soft lower and soft upper value, soft dominated strategies and soft Nash equilibrium. In section 4, we give an application for two person fuzzy soft games. In section 5, we give n-person fuzzy soft games that is extension of the two person fuzzy soft games. In final Section, we concluded the work.

8 Preliminary

In this section, we have introduced the basic definitions of soft sets [8, 34], fuzzy sets [40] and fuzzy soft sets [14] which are useful for subsequent discussions. More detailed explanations related to the soft sets, fuzzy sets and fuzzy soft sets can be found in [2, 8, 15, 30], [40] and [13, 14]), respectively.

Notion of the soft set theory is first given by Molodtsov [34]. Then the definition of soft set is modified by Çağman and Enginoğlu [8] as follows.

Definition 8.1 [8] *Let U be a universe, E be a set of parameters that are describe the elements of U , and $A \subseteq E$. Then, a soft set F_A over U is a set defined by a set valued function f_A representing a mapping*

$$f_A : E \rightarrow P(U) \text{ such that } f_A(x) = \emptyset \text{ if } x \in E - A$$

where f_A is called approximate function of the soft set F_A . Generally, f_A, g_B, h_C, \dots will be used as an approximate functions of F_A, G_B, H_C, \dots , respectively.

The value of approximate function $f(x)$ may be arbitrary, some of them may be empty, some may have nonempty intersection.

It is noting that the soft set is a parametrized family of subsets of the set U , and therefore it can be written a set of ordered pairs

$$F_A = \{(x, f_A(x)) : x \in E\}$$

The subscript A in the f_A indicates that f_A is the approximate function of F_A .

Note that if $f_A(x) = \emptyset$, then the element $(x, f_A(x))$ is not appeared in F_A .

Definition 8.2 [40] Let U be the universe. A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping

$$\mu_X : U \rightarrow [0, 1].$$

The value $\mu_X(x)$ for the fuzzy set X is called the membership value or the grade of membership of $x \in U$. The membership value represents the degree of x belonging to the fuzzy set X . Then a fuzzy set X on U can be represented as follows,

$$X = \{(\mu_X(x)/x) : x \in U, \mu_X(x) \in [0, 1]\}.$$

Definition 8.3 [14] Let U be an initial universe, $F(U)$ be all fuzzy sets over U . E be the set of all parameters and $A \subseteq E$. An fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows,

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}$$

where $\gamma_A : E \rightarrow F(U)$ such that $\gamma_A(x) = \emptyset$ if $x \notin A$, and for all $x \in E$

$$\gamma_A(x) = \{\mu_{\gamma_A(x)}(u)/u : u \in U, \mu_{\gamma_A(x)}(u) \in [0, 1]\}$$

is a fuzzy set over U .

The subscript A in the γ_A indicates that γ_A is the approximate function of Γ_A .

Note that if $\gamma_A(x) = \emptyset$, then the element $(x, \gamma_A(x))$ is not appeared in Γ_A .

Example 8.4 Suppose that $U = \{u_1, u_2, u_3, u_4\}$ is the universe contains four house under consideration in an auto agent and $E = \{x_1, x_2, x_3, x_4\}$ is the set of parameters, where x_i ($i = 1, 2, 3, 4$) stand for ‘garden’, ‘cheap’, ‘modern’ and ‘large’, respectively.

A customer to select a house from the real estate agent, can construct a fuzzy soft set Γ_A that describes the characteristic of houses according to own requests. Assume that $A = \{x_1, x_2, x_3, x_4\} \subseteq E$ and $\gamma_A(x_1) = \{0.4/u_1, 0.3/u_4\}$, $\gamma_A(x_2) = \{0.5/u_2\}$, $\gamma_A(x_3) = \emptyset$, $\gamma_A(x_4) = \{0.2/u_1, 0.8/u_2\}$ then the fuzzy soft-set Γ_A is written by

$$\Gamma_A = \{(x_1, \{0.4/u_1, 0.3/u_4\}), (x_2, \{0.5/u_2\}), (x_4, \{0.2/u_1, 0.8/u_2\})\}$$

By using same parameter set A , another customer to select a house from the same real estate agent, can construct a fuzzy soft set Γ'_A according to own

requests. Here Γ'_A may be different then Γ_A . Assume that $\gamma'_A(x_1) = \emptyset$, $\gamma'_A(x_2) = \{0.3/u_2, 0.5/u_3, 0.1/u_4\}$, $\gamma'_A(x_3) = \{0.4/u_1, 0.4/u_2, 0.9/u_3\}$, $\gamma'_A(x_4) = \{0.7/u_4\}$, then the fuzzy soft set Γ_A is written by

$$\Gamma'_A = \{(x_2, \{0.3/u_2, 0.5/u_3, 0.1/u_4\}), (x_3, \{0.4/u_1, 0.4/u_2, 0.9/u_3\}), (x_4, \{0.7/u_4\})\}$$

Definition 8.5 [14] Let Γ_A and Γ_B be two fuzzy soft sets. Then,

- a) Complement of Γ_A is denoted by Γ_A^c . Its approximate function γ_{A^c} is defined by

$$\gamma_{A^c}(x) = \gamma_A^c(x), \text{ for all } x \in E,$$

- b) Union of Γ_A and Γ_B is denoted by $\Gamma_A \tilde{\cup} \Gamma_B$. Its fuzzy approximate function $\gamma_{A \tilde{\cup} B}$ is defined by

$$\gamma_{A \tilde{\cup} B}(x) = \gamma_A(x) \cup \gamma_B(x) \text{ for all } x \in E.$$

- c) Intersection of Γ_A and Γ_B is denoted by $\Gamma_A \tilde{\cap} \Gamma_B$. Its fuzzy approximate function $\gamma_{A \tilde{\cap} B}(x)$ is defined by

$$\gamma_{A \tilde{\cap} B}(x) = \gamma_A(x) \cap \gamma_B(x) \text{ for all } x \in E.$$

- d) Γ_A is an fuzzy soft subset of Γ_B , denoted by $\Gamma_A \tilde{\subseteq} \Gamma_B$, if $\gamma_A(x) \subseteq \gamma_B(x)$ for all $x \in E$.

9 Two Person Fuzzy Soft Games

In this section, we construct two person fuzzy soft games with fuzzy soft pay-offs. We then give four solution methods for the fuzzy soft games. The basic definitions and preliminaries of the soft set, game and soft game theory we refer to [11, 15, 34].

In the soft game[11], the strategy sets and the soft payoffs are crisp. But in fuzzy soft game, while the strategy sets are crisp, the fuzzy soft payoffs are fuzzy subsets of U . To avoid the confusion we will use $\Gamma_A^k, \Gamma_B^k, \Gamma_C^k, \dots$, etc. for two person fuzzy soft game and $\gamma_A^k, \gamma_B^k, \gamma_C^k, \dots$, etc. for their fuzzy soft payoffs, respectively.

Definition 9.1 Let E be a set of strategy and $X, Y \subseteq E$. A choice of behaviour in a fuzzy soft game is called an action. The elements of $X \times Y$ are called action pairs. That is, $X \times Y$ is the set of available actions.

Definition 9.2 Let U be a set of alternatives, $F(U)$ be all fuzzy sets over U , E be a set of strategies, $X, Y \subseteq E$. Then, a set valued function

$$\gamma_{X \times Y} : X \times Y \rightarrow F(U)$$

is called a fuzzy soft payoff function. For each $(x, y) \in X \times Y$, the value $\gamma_{X \times Y}(x, y)$ is called a fuzzy soft payoff.

Definition 9.3 Let $X \times Y$ be a set of action pairs. Then, an action $(x^*, y^*) \in X \times Y$ is called an optimal action if

$$\gamma_{X \times Y}(x^*, y^*) \supseteq \gamma_{X \times Y}(x, y) \text{ for all } (x, y) \in X \times Y.$$

Definition 9.4 Let $X \times Y$ be a set of action pairs and $(x_i, y_j), (x_r, y_s) \in X \times Y$. Then,

- a) if $\gamma_{X \times Y}(x_i, y_j) \supset \gamma_{X \times Y}(x_r, y_s)$, we says that a player strictly prefers action pair (x_i, y_j) over action (x_r, y_s) ,
- b) if $\gamma_{X \times Y}(x_i, y_j) = \gamma_{X \times Y}(x_r, y_s)$, we says that a player is indifferent between the two actions,
- c) if $\gamma_{X \times Y}(x_i, y_j) \supseteq \gamma_{X \times Y}(x_r, y_s)$, we says that a player either prefers (x_i, y_j) to (x_r, y_s) or is indifferent between the two actions.

Definition 9.5 Let $\gamma_{X \times Y}^k$ be a fuzzy soft payoff for Player k , ($k = 1, 2$), and $(x_i, y_j), (x_r, y_s) \in X \times Y$. Then, Player k is called rational, if the player's fuzzy soft payoff satisfies the following conditions:

- a) Either $\gamma_{X \times Y}^k(x_i, y_j) \supseteq \gamma_{X \times Y}^k(x_r, y_s)$ or $\gamma_{X \times Y}^k(x_r, y_s) \supseteq \gamma_{X \times Y}^k(x_i, y_j)$
- b) If $\gamma_{X \times Y}^k(x_i, y_j) \supseteq \gamma_{X \times Y}^k(x_r, y_s)$ and $\gamma_{X \times Y}^k(x_r, y_s) \supseteq \gamma_{X \times Y}^k(x_i, y_j)$, then $\gamma_{X \times Y}^k(x_i, y_j) = \gamma_{X \times Y}^k(x_r, y_s)$.

Definition 9.6 Let X and Y be a set of strategies of Player 1 and 2, respectively, U be a set of alternatives and $\gamma_{X \times Y}^k : X \times Y \rightarrow F(U)$ be a fuzzy soft payoff function for player k , ($k = 1, 2$). Then, for each Player k , a two person fuzzy soft game (tpfs-game) is defined by a fuzzy soft set over U as

$$\Gamma_{X \times Y}^k = \{((x, y), \gamma_{X \times Y}^k(x, y)) : (x, y) \in X \times Y\}$$

The tpfs-game is played as follows: at a certain time Player 1 chooses a strategy $x_i \in X$, simultaneously Player 2 chooses a strategy $y_j \in Y$ and once this is done each player k ($k=1,2$) receives the fuzzy soft payoff $\gamma_{X \times Y}^k(x_i, y_j)$.

If $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_n\}$, then the fuzzy soft payoffs of $\Gamma_{X \times Y}^k$ can be arranged in the form of the $m \times n$ matrix shown in Table 1.

$\Gamma_{X \times Y}^k$	y_1	y_2	\dots	y_n
x_1	$\gamma_{X \times Y}^k(x_1, y_1)$	$\gamma_{X \times Y}^k(x_1, y_2)$	\dots	$\gamma_{X \times Y}^k(x_1, y_n)$
x_2	$\gamma_{X \times Y}^k(x_2, y_1)$	$\gamma_{X \times Y}^k(x_2, y_2)$	\dots	$\gamma_{X \times Y}^k(x_2, y_n)$
\vdots	\vdots	\vdots	\ddots	\vdots
x_m	$\gamma_{X \times Y}^k(x_m, y_1)$	$\gamma_{X \times Y}^k(x_m, y_2)$	\dots	$\gamma_{X \times Y}^k(x_m, y_n)$

Table 1: The two person fuzzy soft game

Now, we can give an example for tpfs-game.

Example 9.7 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ be a set of alternatives, $F(U)$ be all fuzzy sets over U , $E = \{x_1, x_2, x_3, x_4, x_5\}$ be a set of strategies and $X = \{x_1, x_2, x_4\}$ and $Y = \{x_1, x_2\}$ be a set of the strategies Player 1 and 2, respectively.

If Player 1 constructs a tpfs-games as follows,

$$\Gamma_{X \times Y}^1 = \left\{ ((x_1, x_1), \{0.7/u_1, 0.6/u_2, 0.4/u_5\}), (x_1, x_2), \{0.2/u_1, 0.3/u_2, 0.8/u_3, 0.1/u_4, 0.9/u_5\}), ((x_3, x_1), \{0.8/u_1, 0.1/u_3\}), (x_3, x_2), \{0.5/u_1, 0.3/u_2, 0.8/u_3, 0.7/u_5\}), ((x_5, x_1), \{0.5/u_3, 0.7/u_5, 0.3/u_4\}), (x_5, x_2), \{0.5/u_1, 0.6/u_2, 0.5/u_3, 0.7/u_4, 0.3/u_5\}) \right\}$$

then the fuzzy soft payoffs of the game can be arranged as in Table 2,

$\Gamma_{X \times Y}^1$	x_1	x_2
x_1	$\{0.7/u_1, 0.6/u_2, 0.4/u_5\}$	$\{0.2/u_1, 0.3/u_2, 0.8/u_3, 0.1/u_4, 0.9/u_5\}$
x_3	$\{0.8/u_1, 0.1/u_3\}$	$\{0.5/u_1, 0.3/u_2, 0.8/u_3, 0.7/u_5\}$
x_5	$\{0.5/u_3, 0.7/u_5, 0.3/u_4\}$	$\{0.5/u_1, 0.6/u_2, 0.5/u_3, 0.7/u_4, 0.3/u_5\}$

Table 2

Let us explain some element of this game; if Player 1 select x_3 and Player 2 select x_2 , then the value of game will be a fuzzy soft payoff $\gamma_{X \times Y}^1(x_3, x_2) = \{0.5/u_1, 0.3/u_2, 0.8/u_3, 0.7/u_5\}$. In this case, Player 1 wins the set of alternatives $\{0.5/u_1, 0.3/u_2, 0.8/u_3, 0.7/u_5\}$ and Player 2 lost the same set of alternatives.

Similarly, if Player 2 constructs a tpfs-game as follows,

$$\Gamma_{X \times Y}^2 = \left\{ ((x_1, x_1), \{0.7/u_3, 0.6/u_4, 0.4/u_6\}), (x_1, x_2), \{0.2/u_6\}), ((x_3, x_1), \{0.8/u_2, 0.1/u_4, 0.3/u_5, 0.8/u_6\}), (x_3, x_2), \{0.5/u_4, 0.3/u_6\}), ((x_5, x_1), \{0.5/u_1, 0.7/u_2, 0.3/u_6\}), (x_5, x_2), \{0.5/u_6\}) \right\}$$

then the fuzzy soft payoffs of the game can be arranged as in Table 3,

$\Gamma_{X \times Y}^2$	x_1	x_2
x_1	$\{0.7/u_3, 0.6/u_4, 0.4/u_6\}$	$\{0.2/u_6\}$
x_3	$\{0.8/u_2, 0.1/u_4, 0.3/u_5, 0.8/u_6\}$	$\{0.5/u_4, 0.3/u_6\}$
x_5	$\{0.5/u_1, 0.7/u_2, 0.3/u_6\}$	$\{0.5/u_6\}$

Table 3

Let us explain some element of this tpfs-game; if Player 1 select x_3 and Player 2 select x_2 , then the value of game will be fuzzy soft payoff $\gamma_{X \times Y}^2(x_3, x_2) = \{0.5/u_4, 0.3/u_6\}$. In this case, Player 1 wins the set of alternatives $\{u_4, u_8\}$ and Player 2 lost $\{0.5/u_4, 0.3/u_6\}$.

Now the two person zero sum game on the classical game theory will be a two person empty intersection game on the fuzzy soft game theory. It is given in following definition.

Definition 9.8 A *tpfs-game* is called a two person empty intersection fuzzy soft game if intersection of the fuzzy soft payoff of players is empty set for each action pairs.

For instance, Example 9.7 is a two person empty intersection fuzzy soft game.

Definition 9.9 Let $\gamma_{X \times Y}^k$ be a fuzzy soft payoff function of a *tpfs-game* $\Gamma_{X \times Y}^k$. If the following properties hold

$$a) \bigcup_{i=1}^m \gamma_{X \times Y}^k(x_i, y_j) = \gamma_{X \times Y}^k(x, y)$$

$$b) \bigcap_{j=1}^n \gamma_{X \times Y}^k(x_i, y_j) = \gamma_{X \times Y}^k(x, y)$$

then $\gamma_{X \times Y}^k(x, y)$ is called a fuzzy soft saddle point of Player k 's in the *tpfs-game*.

Note that if $\gamma_{X \times Y}^1(x, y)$ is a fuzzy soft saddle point of a *tpfs-game* $\Gamma_{X \times Y}^1$, then Player 1 can then win at least by choosing the strategy $x \in X$, and Player 2 can keep her/his loss to at most $\gamma_{X \times Y}^1(x, y)$ by choosing the strategy $y \in Y$. Hence the fuzzy soft saddle point is a value of the *tpfs-game*.

Example 9.10 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the strategies for Player 1 and 2, respectively. Then, *tpfs-game* of Player 1 is given as in Table 4,

$\Gamma_{X \times Y}^1$	y_1	y_2
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$	$\{0.9/u_1, 0.6/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$	$\{0.5/u_1, 0.8/u_3, 0.8/u_5\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$	$\{0.5/u_1, 0.5/u_3, 0.7/u_4\}$

Table 4

Clearly,

$$\begin{aligned} \bigcup_{i=1}^4 \gamma_{X \times Y}^1(x_i, y_1) &= \{0.8/u_1, 0.4/u_2, 0.6/u_4\}, \\ \bigcup_{i=1}^4 \gamma_{X \times Y}^1(x_i, y_2) &= \{0.9/u_1, 0.6/u_2, 0.8/u_3, 0.9/u_4, 0.8/u_5\} \end{aligned}$$

and

$$\begin{aligned} \bigcap_{j=1}^3 \gamma_{X \times Y}^1(x_1, y_j) &= \{0.8/u_1, 0.4/u_2, 0.6/u_4\}, \\ \bigcap_{j=1}^3 \gamma_{X \times Y}^1(x_2, y_j) &= \{0.5/u_1\}, \\ \bigcap_{j=1}^3 \gamma_{X \times Y}^1(x_3, y_j) &= \{0.2/u_1, 0.1/u_4\}. \end{aligned}$$

Therefore, $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$ is a fuzzy soft saddle point of the *tpfs-game*, since the intersection of the forth row is equal to the union of the third column. So, the value of the *tpfs-game* is $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

Note that every *tpfs*-game has not a fuzzy soft saddle point. (For instance, in the above example, if $\{0.8/u_1, 0.4/u_2\}$ is replaced with $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$ in fuzzy soft payoff $\gamma_{X \times Y}^1(x_1, y_1)$, then a fuzzy soft saddle point of the game can not be found.) Saddle point can not be used for a *tpfs*-game, fuzzy soft upper and fuzzy soft lower values of the *tpfs*-game may be used is given in the following definition.

Definition 9.11 Let $\Gamma_{X \times Y}$ be a *tpfs*-game with its fuzzy soft payoff function $\gamma_{X \times Y}$. Then,

- i. Fuzzy soft upper value of the *tpfs*-game, denoted \bar{v} , is defined by

$$\bar{v} = \bigcap_{y \in Y} (\bigcup_{x \in X} (\gamma_{X \times Y}(x, y)))$$

- ii. Fuzzy soft lower value of the *tpfs*-game, denoted \underline{v} , is defined by

$$\underline{v} = \bigcup_{x \in X} (\bigcap_{y \in Y} (\gamma_{X \times Y}(x, y)))$$

- iii. If fuzzy soft upper and fuzzy soft lower value of a *tpfs*-game are equal, they are called value of the *tpfs*-game, noted by v . That is $v = \underline{v} = \bar{v}$.

Example 9.12 Let us consider Table 4 in Example 9.10. It is clear that fuzzy soft upper value $\bar{v} = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$ and fuzzy soft lower value $\underline{v} = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$, hence $\underline{v} = \bar{v}$. It means that value of the *tpfs*-game is $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

Theorem 9.13 \underline{v} and \bar{v} be a fuzzy soft lower and fuzzy soft upper value of a *tpfs*-game, respectively. Then, the fuzzy soft lower value is subset or equal to the fuzzy soft upper value, that is,

$$\underline{v} \subseteq \bar{v}$$

Example 9.14 Let us consider fuzzy soft upper value \bar{v} and fuzzy soft lower value \underline{v} in Example 9.12. It is clear that $\bar{v} = \{0.8/u_1, 0.4/u_2, 0.6/u_4\} \subseteq \underline{v} = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$, hence $\underline{v} \subseteq \bar{v}$.

Theorem 9.15 Let $\gamma_{X \times Y}(x, y)$ be a fuzzy soft saddle point, \underline{v} be a fuzzy soft lower value and \bar{v} be a fuzzy soft upper value of a *tpfs*-game. Then,

$$\underline{v} \subseteq \gamma_{X \times Y}(x, y) \subseteq \bar{v}$$

Corollary 9.16 Let $\gamma_{X \times Y}(x, y)$ be a fuzzy soft saddle point, \underline{v} be a fuzzy soft lower value and \bar{v} be a fuzzy soft upper value of a *tpfs*-game. If $v = \underline{v} = \bar{v}$, then $\gamma_{X \times Y}(x, y)$ is exactly v .

Example 9.17 Let us consider Table 4 in Example 9.10 and fuzzy soft upper value \bar{v} and fuzzy soft lower value \underline{v} in Example 9.12. It is clear that fuzzy soft saddle point $\gamma_{X \times Y}(x, y)$ is exactly $v = \underline{v} = \bar{v} = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

Note that in every *tpfs*-game, the fuzzy soft lower value \underline{v} can not be equals to the fuzzy soft upper value \overline{v} . (For instance, in the above example, if fuzzy soft payoff $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$ is replaced with $\{0.1/u_1, 0.4/u_2, 0.6/u_4\}$ in fuzzy soft payoff $\gamma_{X \times Y}^1(x_1, y_1)$, then the fuzzy soft lower value \underline{v} can not be equals to the fuzzy soft upper value \overline{v} .) If in a *tpfs*-game $\underline{v} \neq \overline{v}$, then to get the solution of the game fuzzy soft dominated strategy may be used. We define fuzzy soft dominated strategy for *tpfs*-game as follows.

Definition 9.18 Let $\Gamma_{X \times Y}^1$ be a *tpfs*-game with its fuzzy soft payoff function $\gamma_{X \times Y}^1$. Then,

- a) a strategy $x_i \in X$ is called a fuzzy soft dominated to another strategy $x_r \in X$, if $\gamma_{X \times Y}^1(x_i, y) \supseteq \gamma_{X \times Y}^1(x_r, y)$ for all $y \in Y$,
- b) a strategy $y_j \in Y$ is called a fuzzy soft dominated to another strategy $y_s \in Y$, if $\gamma_{X \times Y}^1(x, y_j) \subseteq \gamma_{X \times Y}^1(x, y_s)$ for all $x \in X$

By using fuzzy soft dominated strategy, *tpfs*-games may be reduced by deleting rows and columns that are obviously bad for the player who uses them. This process of eliminating fuzzy soft dominated strategies sometimes leads us to a solution of a *tpfs*-game. Such a method of solving *tpfs*-game is called a fuzzy soft elimination method.

The following *tpfs*-game can be solved by using the method.

Example 9.19 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the strategies for Player 1 and 2, respectively. Then, *tpfs*-game of Player 1 is given as in Table 5,

$\Gamma_{X \times Y}^1$	y_1	y_2
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$	$\{0.9/u_1, 0.6/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$	$\{0.5/u_1, 0.8/u_2, 0.8/u_5\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$	$\{0.5/u_1, 0.5/u_3, 0.7/u_4\}$

Table 5

The last column is dominated by the light column. Deleting the last column we can obtain Table 6 as:

$\Gamma_{X \times Y}^1$	y_1
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$

Table 6

Now, in Table 6, the bottom and middle row is dominated by the top row. (Note that this is not the case in Table 5). Deleting the bottom and middle row we obtain Table 7 as:

$\Gamma_{X \times Y}^1$	y_1
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$

Table 7

The solution using the method is (x_1, y_1) , that is, value of the tpfs-game is $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

Note that the fuzzy soft elimination method cannot be used for some tpfs-games which do not have a fuzzy soft dominated strategies. In this case, we can use fuzzy soft Nash equilibrium that is defined as follows.

Definition 9.20 Let $\Gamma_{X \times Y}^k$ be a tpfs-game with its fuzzy soft payoff function $\gamma_{X \times Y}^k$ for $k = 1, 2$. If the following properties hold

a) $\gamma_{X \times Y}^1(x^*, y^*) \supseteq \gamma_{X \times Y}^1(x, y^*)$ for each $x \in X$

b) $\gamma_{X \times Y}^2(x^*, y^*) \supseteq \gamma_{X \times Y}^2(x^*, y)$ for each $y \in Y$

then, $(x^*, y^*) \in X \times Y$ is called a fuzzy soft Nash equilibrium of a tpfs-game.

Note that if $(x^*, y^*) \in X \times Y$ is a fuzzy soft Nash equilibrium of a tpfs-game, then Player 1 can then win at least $\gamma_{X \times Y}^1(x^*, y^*)$ by choosing strategy $x^* \in X$, and Player 2 can win at least $\gamma_{X \times Y}^2(x^*, y^*)$ by choosing strategy $y^* \in Y$. Hence the fuzzy soft Nash equilibrium is an optimal action for tpfs-game, therefore, $\gamma_{X \times Y}^k(x^*, y^*)$ is the solution of the tpfs-game for Player k , $k = 1, 2$.

Following game, given in Example 9.21, can be solved by fuzzy soft Nash equilibrium, but it is very difficult to solve by using the others methods.

Example 9.21 Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}\}$ be a set of alternatives, $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the strategies Player 1 and 2, respectively. Then, tpfs-game of Player 1 is given as in Table 8,

$\Gamma_{X \times Y}^1$	y_1	y_2
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$	$\{0.6/u_1, 0.7/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$	$\{0.5/u_1, 0.8/u_2, 0.8/u_5\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$	$\{0.5/u_1, 0.5/u_3, 0.7/u_4\}$

Table 8

and tpfs-game of Player 2 is given as in Table 9,

$\Gamma_{X \times Y}^1$	y_1	y_2
x_1	$\{0.9/u_1, 0.6/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$
x_2	$\{0.5/u_1, 0.8/u_2, 0.8/u_5\}$	$\{0.7/u_1, 0.1/u_2\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$	$\{0.5/u_1, 0.5/u_3, 0.7/u_4\}$

Table 9

From the tables, we have

a) $\gamma_{X \times Y}^1(x_1, y_1) \supseteq \gamma_{X \times Y}^1(x, y_1)$ for each $x \in X$, and

b) $\gamma_{X \times Y}^2(x_1, y_1) \supseteq \gamma_{X \times Y}^2(x_1, y)$ for each $y \in Y$

then, $(x_1, y_1) \in X \times Y$ is a fuzzy soft Nash equilibrium. Therefore, $\gamma_{X \times Y}^1(x_1, y_1) = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$ and $\gamma_{X \times Y}^2(x_1, y_1) = \{0.9/u_1, 0.6/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$ are the solution of the tpfs-game for Player 1 and Player 2, respectively.

10 An Application

In this section, we give a financial problem that are solved by using both fuzzy soft dominated strategy and fuzzy soft saddle point methods. Now, we modified the application in [11] by using fuzzy soft set as follows;

There are two companies, say Player 1 and Player 2, who competitively want to increase sale of produces in the country. Therefore, they give advertisements. Assume that two companies have a set of different products $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ where for $i = 1, 2, \dots, 8$, the product u_i stand for “oil”, “salt”, “honey”, “jam”, “cheese”, “sugar”, “cooker”, and “jar”, respectively. The products can be characterized by a set of strategy $E = \{x_i : i = 1, 2, 3\}$ which contains styles of advertisement where for $j = 1, 2, 3$, the strategies x_j stand for “TV”, “radio” and “newspaper”, respectively.

Suppose that $X = \{x_1, x_2, x_3\}$ and $Y = \{x_1, x_2\}$ are strategies of Player 1 and 2, respectively. Then, a *tpfs*-game of Player 1 is given as in Table 10.

$\Gamma_{X \times Y}^1$	x_1	x_2
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$	$\{0.9/u_1, 0.7/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$	$\{0.7/u_1, 0.8/u_4, 0.8/u_5\}$
x_3	$\{0.2/u_1, 0.1/u_4\}$	$\{0.5/u_1, 0.5/u_3, 0.7/u_4\}$

Table 10

In Table 10, let us explain action pair (x_1, x_1) ; if Player 1 select $x_1 = \text{“TV”}$ and Player 2 select $x_1 = \text{“TV”}$, then the fuzzy soft payoff of Player 1 is a set $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$, that is, $\gamma_{X \times Y}^1(x_1, x_1) = \{0.8/u_1, 0.4/u_2, 0.6/u_4\}$. In this case, Player 1 increase sale of $\{u_1, u_2, u_3, u_5, u_8\}$ and Player 2 decrease sale of $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

We can now solve the game. It is seen in Table 10,

$$\begin{aligned} \{0.8/u_1, 0.4/u_2, 0.6/u_4\} &\subseteq \{0.2/u_1, 0.1/u_4\} \\ \{0.9/u_1, 0.7/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\} &\subseteq \{0.5/u_1, 0.5/u_3, 0.7/u_4\} \end{aligned}$$

the bottom row is dominated by the top row. We then deleting the bottom row we obtain Table 11.

$\Gamma_{X \times Y}^1$	x_1	x_2
x_1	$\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$	$\{0.9/u_1, 0.7/u_2, 0.6/u_3, 0.9/u_4, 0.5/u_5\}$
x_2	$\{0.7/u_1, 0.1/u_2\}$	$\{0.7/u_1, 0.8/u_4, 0.8/u_5\}$

Table 11

In Table 11, there is no another fuzzy soft dominated strategy, we can use fuzzy

soft saddle point method.

$$\begin{aligned}
\bigcup_{i=1}^2 \gamma_{X \times Y}^1(x_i, x_1) &= \{0.8/u_1, 0.4/u_2, 0.6/u_4\} \\
\bigcup_{i=1}^2 \gamma_{X \times Y}^1(x_i, x_2) &= \{0.7/u_1, 0.7/u_2, 0.6/u_3, 0.9/u_4, 0.8/u_5\} \\
\bigcap_{j=1}^2 \gamma_{X \times Y}^1(x_1, y_j) &= \{0.8/u_1, 0.4/u_2, 0.6/u_4\} \\
\bigcap_{j=1}^2 \gamma_{X \times Y}^1(x_2, y_j) &= \{0.7/u_1\}
\end{aligned}$$

Here, optimal strategy of the game is (x_1, y_1) since

$$\bigcup_{i=1}^2 \gamma_{X \times Y}^1(x_i, y_1) = \bigcap_{j=1}^2 \gamma_{X \times Y}^1(x_1, y_j)$$

Therefore, value of the *tpfs*-game is $\{0.8/u_1, 0.4/u_2, 0.6/u_4\}$.

11 n -Person Soft Games

In many applications the fuzzy soft games can be often played between more than two players. Therefore, *tpfs*-games can be extended to n -person fuzzy soft games.

From now on, X_n^\times will be used for $X_1 \times X_2 \times \dots \times X_n$.

Definition 11.1 Let U be a set of alternatives, $F(U)$ be all fuzzy sets over U , E be a set of strategies, $X_1, X_2, \dots, X_n \subseteq E$, and X_k is the set of strategies of Player k , ($k = 1, 2, \dots, n$). Then, for each Player k , an n -person fuzzy soft game (*nps-game*) is defined by a fuzzy soft set over U as

$$\Gamma_{X_n^\times}^k = \{((x_1, x_2, \dots, x_n), \gamma_{X_n^\times}^k(x_1, x_2, \dots, x_n)) : (x_1, x_2, \dots, x_n) \in X_n^\times\}$$

where $\gamma_{X_n^\times}^k$ is a fuzzy soft payoff function of Player k .

The *nps*-game is played as follows: at a certain time Player 1 chooses a strategy $x_1 \in X_1$ and simultaneously each Player k ($k = 2, \dots, n$) chooses a strategy $x_k \in X_k$ and once this is done each player k receives the fuzzy soft payoff $\gamma_{X_n^\times}^k(x_1, x_2, \dots, x_n)$.

Definition 11.2 Let $\Gamma_{X_n^\times}^k$ be an *nps*-game with its fuzzy soft payoff function $\gamma_{X_n^\times}^k$ for $k = 1, 2, \dots, n$. Then, a strategy $x_k \in X_k$ is called a fuzzy soft dominated to another strategy $x \in X_k$, if

$$\gamma_{X_n^\times}^k(x_1, \dots, x_{k-1}, x_k, x_{k+1}, \dots, x_n) \supseteq \gamma_{X_n^\times}^k(x_1, \dots, x_{k-1}, x, x_{k+1}, \dots, x_n)$$

for each strategy $x_i \in X_i$ of player i ($i = 1, 2, \dots, k-1, k+1, \dots, n$), respectively.

Definition 11.3 Let $\gamma_{X_n^\times}^k$ be a fuzzy soft payoff function of a nps-game $\Gamma_{X_n^\times}^k$. If for each player k ($k=1,2,\dots,n$) the following properties hold

$$\gamma_{X_n^\times}^k(x_1^*, \dots, x_{k-1}^*, x_k^*, x_{k+1}^*, \dots, x_n^*) \supseteq \gamma_{X_n^\times}^k(x_1^*, \dots, x_{k-1}^*, x, x_{k+1}^*, \dots, x_n^*)$$

for each $x \in X_k$, then $(x_1^*, x_2^*, \dots, x_n^*) \in X_n^\times$ is called a fuzzy soft Nash equilibrium of an nps-game.

12 Conclusion

In this paper, we first present the basic definitions and results of fuzzy soft set theory. We then construct *tpfs*-games with fuzzy soft payoffs which is set value and the solution operations based on the set operations. We also give four solution methods for the *tpfs*-games with examples. To applied the game to the real world problem we give an application which shows the methods can be successfully applied to a financial problem. Finally, we extended the two person fuzzy soft games to n -person fuzzy soft games. The fuzzy soft games may be applied to many fields and more comprehensive in the future to solve the related problems, such as; computer science, decision making, and so on.

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